

Faculty of Sciences
Moulay Ismail University - Meknès
Laboratory of Pure Mathematics

J.A.M 2025

4ÈMES JOURNÉES ARITHMÉTIQUES DE MEKNÈS
International Conference on Number Theory and
Applications

Book of Abstracts

April 14 - 18, 2025



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Introduction

The “Journées Arithmétiques de Meknès” (J.A.M) is an international conference dedicated to recent advances in number theory and its applications. The fourth edition of this event, JAM 2025, is organized by the Laboratory of Pure Mathematics at the Faculty of Sciences, Moulay Ismail University in Meknès, Morocco. It will take place from April 14 to 18, 2025, with a special session hosted at ENSAM Meknès on April 17, 2025.

This conference provides a stimulating environment for researchers from across the globe to engage in fruitful discussions on a wide range of topics in arithmetic geometry, algebraic and analytic number theory, modular forms, Iwasawa theory, Galois representations, cryptography, and related fields. By bringing together senior experts and young researchers, JAM 2025 fosters the exchange of ideas, the development of collaborations, and the presentation of recent results and open problems in number theory.

This ** Book of Abstracts** gathers the summaries of the talks presented during the conference. It reflects the diversity and depth of the mathematical contributions, including both theoretical explorations and concrete applications. We are pleased to welcome participants from a variety of academic institutions worldwide, highlighting the truly international character of the event.

The Organizing Committee warmly thanks all the speakers and contributors for their participation and enthusiasm. We are particularly grateful to our scientific committee, institutional partners, and sponsors, whose support made this conference possible.

We wish all attendees an inspiring and productive week in Meknès, and hope this event will strengthen connections and lead to new mathematical discoveries.

The Organizing Committee
JAM 2025

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- Michel WALDSCHMIDT, Sorbonne University, France.
- Abdelkader ZEKHNINI, Mohammed First University, FS, Oujda, Morocco.

Speakers and Abstracts

This section contains the list of contributed talks and abstracts presented during the JAM 2025 conference.

Each abstract includes the speaker's name, affiliation, and a summary of their presentation in the field of number theory and its applications.

Information Geometry for Channel Capacity

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Abstract

In this work, we propose an information geometry approach (IGA) for channel capacity, we introduced the new algorithm that monotonically increases the Kullback-Leibler divergence. We calculate the approximation of the a posteriori information, it is formulated as an iterative m-projection and e-projection process between submanifolds with different constraints. We apply the information geometry to simplify the calculation of the m-projection and e-projection since the direct calculation is difficult.

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Transcendence of some p -adic continued fractions

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Joint work with: Pr. Kacha Ali

Abstract

In this contribution, we establish sufficient conditions on the elements of the p -adic continued fractions A and B which guarantee that the continued fraction A^B is a transcendental number in the p -adic framework.

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Structure of the automorphism group of Danielewski-type surfaces

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Joint work with: M'hammed El Kahoui

Abstract

Danielewski surfaces were introduced in [3] as a counterexample to the Zariski Cancellation Problem over algebraically closed fields. These are affine algebraic surfaces embedded in the affine 3-space $\mathbb{A}_{\mathbb{C}}^3$ and defined by an equation of the form

$$x^n z - q(y) = 0,$$

where $n \in \mathbb{N}^*$ and $q(y) \in \mathbb{C}[y]$ has degree at least two. Since then, several Danielewski-type surfaces have been introduced and studied in many different contexts, see e.g., [1, 2, 4–9], in particular their K -automorphisms groups were described.

In this talk, we propose a general and purely algebraic method to describe the K -automorphism group of an arbitrary Danielewski-type surface. Specifically, we consider K -algebras of the form

$$A_{c,q} = K[x, y, z] / (c(x)z - q(x, y)),$$

where $c(x) \in K[x]$ is a polynomial of degree $n \geq 2$, and $q(x, y) \in K[x, y]$ is a quasi-homonic polynomial of degree $d \geq 2$ with respect to y .

The algebra $A_{c,q}$ is naturally endowed with a locally nilpotent K -derivation $\xi_{c,q}$ defined by $\xi_{c,q}(\bar{y}) = c(x)$ and $\xi_{c,q}(\bar{z}) = \partial_y q(\bar{x}, \bar{y})$. We first prove that $\ker(\xi_{c,q}) = K[\bar{x}]$ and that every locally nilpotent K -derivation of $A_{c,q}$ has the form $a\xi_{c,q}$ for some $a \in K[\bar{x}]$. As a consequence, every automorphism in $\text{Aut}_K(A_{c,q})$ preserves the *plinth* ideal $\text{pl}(\xi_{c,q})$ of $\xi_{c,q}$, leading to the construction of a natural group homomorphism

$$\psi : \text{Aut}_K(A_{c,q}) \longrightarrow K^* \times K^*,$$

The kernel of ψ is the subgroup $\text{SAut}(A_{c,q})$ of *exponential* K -automorphisms of $A_{c,q}$, yielding the short exact sequence

$$1 \longrightarrow \text{SAut}(A_{c,q}) \longrightarrow \text{Aut}_K(A_{c,q}) \xrightarrow{\psi} \text{Im}(\psi) \longrightarrow 1.$$

One of our main results asserts that this sequence is split exact, implying that $\text{Aut}_K(A_{c,q})$ is the *inner* semidirect product of $\text{SAut}(A_{c,q})$ and a subgroup of $K^* \times K^*$, which we explicitly describe in terms of $c(x)$ and $q(x, y)$. Moreover, we establish that this subgroup has only five possibilities. Finally, we present an algorithm that determines precisely which of these five possibilities occurs for a given input data $c(x)$ and $q(x, y)$.

Keywords: Danielewski algebra, Locally nilpotent derivation.

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The 2-Part of Class Group of Pure Quartic Number Fields

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Joint work with: Pr. Mohammed TAOUS

Abstract

let $K = \mathbb{Q}(\sqrt[4]{m})$ denote a real pure quartic field, where m is a natural number, Cl_K its class group, h_K its class number and k its quadratic subfield. Using the fact that $\mathbb{Q}(\sqrt[4]{m}) = \mathbb{Q}(\sqrt[4]{m^3})$, it is sufficient and without loss of generality to write K in the form $K = \mathbb{Q}(\sqrt[4]{ab^2c^3})$, where $a > 1$, and b, c are natural numbers that are square-free and pairwise coprime, with c being odd. Consequently, the quadratic subfield is given by $k = \mathbb{Q}(\sqrt{ac})$. In this presentation, we determine the 2-rank of Cl_K in the case where h_k is odd, following the above notations. Furthermore, in the same setting, we present some methods to determine the 4-rank of Cl_K when the Sylow 2-subgroup of Cl_K , denoted by $syl_2(Cl_K)$, is of type $(2, 2)$.

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Generalized Sato-Tate conjecture, motivation and arithmetic consequences

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Abstract

In this talk, we explore a generalized version of the Sato-Tate conjecture for higher-dimensional abelian varieties, motivated by the Langlands program. We will discuss how recent advances in potential automorphy combined with Brauer's induction theorem, provide a framework for approaching it in several cases. Specific results for abelian surfaces potentially of GL₂-type will be highlighted, along with implications for related conjectures in arithmetic geometry.

On the unramified abelian iwasawa module of some number fields

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Joint work with: Ali Mouhib.

Abstract

Let p, q and q' be three distinct odd prime numbers, satisfying certain congruences. We give the structure of the unramified abelian Iwasawa module $X_\infty(\mathbb{Q}(\sqrt{pqq'}))$ of the number field $\mathbb{Q}(\sqrt{pqq'})$.

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Séries d'Eisenstein associées aux caractères de Legendre et les congruences de Ramanujan

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Résumé

Nous présentons ici deux types de séries d'Eisenstein associées aux caractères de Legendre, et nous démontrons qu'il est possible de les exprimer en termes de la fonction η de Dedekind. Nous établissons plusieurs identités fondamentales, chacune reliant des sommes sur les diviseurs d'un entier naturel non nul à des combinaisons de quotients de fonctions η . Cette démarche nous permet d'obtenir d'importantes nouvelles équations modulaires, qui revêtent un intérêt significatif en arithmétique, notamment en lien avec la théorie des partitions. Les outils fondamentaux mobilisés proviennent de la théorie des formes modulaires et de la géométrie des sous-groupes de congruences, de niveau donné, du groupe modulaire $SL_2(\mathbb{Z})$. Grâce à cette approche, nous parvenons à améliorer considérablement les résultats déjà connus concernant les congruences de Ramanujan relatives à la fonction de partition d'un entier naturel. Nos résultats ouvrent ainsi de nouvelles perspectives dans l'étude des propriétés arithmétiques des partitions, dans un cadre plus général et enrichissant.

Keywords: Eisenstein series, L -function, partition functions, Eta function, Legendre symbols, divisors functions, modular functions, Hauptmodul.

subclass [2020]{11M36, 11F20, 11P84, 11T24}

Hiding Codes into Graphs

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Joint Work with Khalid Abdelmoumen.

Abstract

We use augmentation of circulation codes to investigate the possibility of the construction of secure McEliece-type cryptosystems. This work is based on a transformation introduced by Jungnickel and Vanstone in 1999, which we call a ksi function. We can think of the JV-transformation as a general mechanism injecting codes in the algebra of paths of an appropriate digraph. Furthermore, we examine the effect of the ksi function on some algebraic lattices induced by self-dual error-correcting codes.

Keywords: *coding theory, cryptography, lattices.*

References

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On antimatter group algebras

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Joint work with: Said El Baghdadi

Abstract

An integral domain with no atoms is called an antimatter domain. Here, we investigate some irreducibility criteria in $K[G] = \{\sum_i a_i X^{g_i} | a_i \in K \text{ and } g_i \in G\}$. In addition, we characterize the antimatter property for group algebras $K[G]$.

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Mumford curves, automorphic forms and applications

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Joint work with: Pr. Hassan OUKHABA

Abstract

We fix a global function field K , let K_∞ be the completion of K at a place ∞ of K and K_∞^{sep} be the separable closure of K_∞ . Let B an indefinite quaternion algebra over K . We introduce an automorphic representation of the group $G(\mathbb{A}_f) \times \text{Gal}(K_\infty^{sep}/K_\infty)$, where G denotes the algebraic group $B^\times$ and $\mathbb{A}_f$ denotes the finite adeles of K . This representation is obtained from the first ℓ -adic étale cohomology groups on the projective system of a Shimura-Mumford curves S_K , with K an open compact subgroup of $G(\mathbb{A}_f)$. Furthermore, we present the action of Hecke operators on automorphic forms with respect to K . The Hecke algebra is commutative, there exists a basis of common eigenforms and the eigenvalues are algebraic integers. This allows us to attach a Galois representation

$$\rho_{f,\ell} : \text{Gal}(K^{sep}/K) \rightarrow GL_2(\overline{\mathbb{Q}_\ell})$$

to any common Hecke eigenform f and prime ℓ , with a given determinant and trace .

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On Euclidean ideal classes in real biquadratic fields

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Abstract

The classical notion of Euclidean domains was generalized to that of Euclidean ideal classes for the first time by H.W. Lenstra in [5]. He also proved that the existence of a Euclidean ideal class ensures that the class group is cyclic and is generated by the Euclidean class. While previous work showed the converse in real biquadratic fields where the class number equals two, we extend their method to demonstrate the existence of a Euclidean ideal class in real biquadratic fields where the class number is a power of two.

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Arithmetical contribution on perfect graphs

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Joint work with: Mohamed Jorf and Lahcen Oukhtite

Abstract

A perfect graph is a graph in which, for every induced subgraph, the chromatic number is equal to the clique number. The class of perfect graphs enables the resolution of certain complex problems in polynomial time. Therefore, determining whether a given graphs class is perfect is of significant interest.

In this talk, I present a new perfect graphs class, called *coprime divisors graphs*. Their vertices correspond to the divisors of a fixed composite number $n > 1$, and two vertices are adjacent if they are coprime. This arithmetical characterization proves highly useful in solving graph coloring problem, maximum clique problem, and maximum independent set problem. Indeed, we provide explicit expressions of the chromatic number, the clique number, and the independent number purely in terms of the arithmetical properties of the integer n ; especially, p -adic valuations and the number of prime divisors.

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Idéaux réduits de quelques corps quartiques purs

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Joint work with: Abdelkader ZEKHNINI

Abstract

Cet exposé est dédié à l'étude des idéaux réduits de certains corps quartiques purs. Pour cela, nous définissons d'abord les différents types de ces corps. Nous rappelons ensuite les notions essentielles de discriminant et de base intégrale propres à un corps quartique pur. Dans un troisième temps, nous traitons l'arithmétique de quelques corps quartiques purs, nous étudions ensuite leur idéaux primitifs et nous terminons par la présentation de leur idéaux réduits.

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Sur la cloture integrale d'un anneau gradué

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Joint work with: Mohammed Charkani

Résumé

Dans cet exposé, nous développons ce que l'on pourrait appeler " **Algèbre commutative graduée** ". Les techniques utilisées diffèrent des techniques existantes car nous restons dans le contexte gradué, c'est-à-dire que nous établissons une théorie sur les anneaux gradués en partant de l'étude des corps gradués. L'objectif est l'étude de la cloture intégrale d'un anneau gradué commutatif intègre gradué par un monoïde simplifiable et sans torsion. On discutera les notions, d'idéal gr-premier, de gr-anneau de valuation et de l'homogénéité d'une extension, d'un corps gradué de groupe de grades commutatif sans torsion.

The IDF property in semi-group rings

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Abstract

An integral domain is said to have the IDF property when every nonzero element of it has a finite number of non associate irreducible divisors. In this talk, we will discuss The IDF property in a semigroup ring $A[\Gamma]$, where A is an integral domain and Γ a cancellative torsion-free monoid.

Keywords: Factorization, Atomic, IDF, FFD, UFD, MCD.

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On the A property of certain rings

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Joint work with: Pr. Samir Bouchiba

Abstract

Let M be a module on a commutative ring R and $Z_R(M)$ be the set of all zero-divisors of R on M . M is called an A-module if each finitely generated ideal contained in $Z_R(M)$ is annihilated by a non-zero element of M . This property was introduced by Quentel in 1971, and since then, it has been studied by many authors under several different names such as property A, property C, F-McCoy property...etc.

In this presentation we cite some examples of A-Modules, while examining how this property interacts with various algebraic tools and constructions, based on the work of D.D.Anderson and Sangmin Chun. Furthermore, we introduce a new property called the I-McCoyness one (where I is a finitely generated ideal with non-zero annihilator on M), which generalizes the latter and enables us to investigate conditions under which the Rees algebra of an ideal satisfies the A property.

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About Linear Codes Heights

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Abstract

Error-correcting codes play a fundamental role in ensuring reliable communication over noisy channels. Among them, linear codes are particularly significant due to their algebraic structure, which facilitates efficient encoding and decoding. These codes are widely used in digital communications, data storage, and cryptographic applications. The aim of this communication is to introduce the new notion related to error-correcting codes, called the height. We show its theoretical properties and its applications to some notions of coding theory. Computer algebra software enable us to give many significant examples.

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On the index divisors and monogeneity of a certain class of polynomials defined by $x^8 + ax^5 + b$

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Abstract

In this paper, we calculate the index of any octic number field K generated by a root α of a monic irreducible trinomial $F(x) = x^8 + ax^5 + b \in \mathbb{Z}[x]$. Our approach is based on Engstrom's results and the factorization of any rational prime in K . In such a way we give a complete answer to Problem 22 of Narkiewicz for this family of octic number fields. In particular, we show that $i(K) \in \{1, 2, 3, 6\}$. As an application of our results, if $i(K) \neq 1$, then K is not monogenic. Also, we give generators of power integral bases in some cases where $i(K) = 1$. Our results are illustrated by some computational examples.

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On The vanishing Of The Twisted Kummer-Leopoldt Constant Of Totally Real Number Fields

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Joint work with: Jilali ASSIM and Zouhair BOUGHADI

Abstract

Let F be a totally real number field and let p be an odd prime. For an odd integer $i \leq -1$, we give an analytical upper bound of the twisted Kummer-Leopoldt constant $\kappa_i(F)$ introduced in [1]. As a by-product of the proof, we provide a characterization of the vanishing of $\kappa_i(F)$ in terms of the value of zeta function at the integer i . We also investigate the triviality of the twisted Kummer-Leopoldt constant in the cyclotomic tower. In particular, we characterize this triviality as an equality of lambda-invariants of some Iwasawa modules.

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Sur la formule explicite de Riemann et la fonction de répartition des nombres premiers

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Résumé

Les formules explicites jouent un rôle central en théorie analytique des nombres, en reliant la distribution des nombres premiers aux propriétés analytiques des fonctions zêta. Dans cet exposé, nous rappelons la formule explicite classique de Riemann ainsi que son interprétation analytique, en insistant particulièrement sur la connexion, déjà établie dans la littérature, entre les zéros non triviaux de la fonction zêta de Riemann et la fonction de répartition des nombres premiers $\pi(x)$. Nous discutons enfin les analogies avec d'autres classes de fonctions analytiques pour lesquelles des formules explicites similaires existent ou peuvent être envisagées.

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Higher special elements and equivariant annihilators of class groups

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Abstract

We explore the theory of higher special elements through the framework of Rubin-Stark units. Our objective is to establish a Gras-type equality that connects these units to the equivariant annihilators of class groups. We demonstrate how this equality is closely tied to fundamental results in the arithmetic theory of leading terms, such as the equivariant Tamagawa number conjecture and the Brumer-Stark conjecture. This presentation is based on joint work with Jilali Assim and Youness Mazigh.

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About Pólya fields and Iwasawa towers

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Joint work with: Abdelkader Zekhnini

Abstract

Let $\mathcal{O}_K$ and C_K be respectively the ring of integers and the class group of a number field K . For each integer $q \geq 2$, denote by $\prod_q(K)$ the product of all the maximal ideals of $\mathcal{O}_K$ with norm q , if these ideals do not exist we set $\prod_q(K) = \mathcal{O}_K$. The Pólya group of K , denoted by $P_o(K)$ is the subgroup of C_K generated by the classes of the ideals $\prod_q(K)$, and K is called a Pólya field if the module of integer-valued polynomials over $\mathcal{O}_K$ has a regular basis. In this presentation we will give some remarks on the Pólya groups of the subfields of $\mathbb{Z}_p$ -extensions of a number field K .

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Dedekind's criterion and monogeneity of a family of relative number fields

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Abstract

Let $L = K(\alpha)$ be an extension of the number field K , where α satisfies the monic irreducible polynomial

$f(x) = x^n - a \in \mathfrak{o}_K[X]$ with $n = p^r$, where p is a prime integer and r be a positive integer and $\mathfrak{o}_K$ is the integral closure of K .

The purpose of this paper is to study the monogeneity of L/K by using a new version of Dedekind's criterion, also we give an integral basis of a family of number field of degree $2p^r$ for some positive integer r . As an illustration, we get a slightly simpler computation of relative discriminant $D_{L/K}$.

Keywords: DVR, Dedekind ring, monogeneity, Relative integral basis

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On Concatenating Three Entanglement Assisted Quantum Codes

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Joint work with: Tadmori Abdelhamid and El Khattabi Hafsa

Abstract

This work investigates the concatenation of three entanglement-assisted quantum error-correcting codes (EAQECCs) derived from cyclic codes over specific rings R . We systematically analyze how the order of concatenating three codes (inner, middle, and outer) influences critical parameters such as pre-shared entanglement consumption, logical error probabilities, and pseudo-thresholds. By establishing that cyclic codes over R , when optimized to saturate the Griesmer, Plotkin, and Singleton bounds, yield EAQECCs that correspondingly saturate quantum bounds, we develop explicit families of concatenated codes with provably optimal performance. The algebraic richness of R , combined with multi-layered redundancy, enhances error suppression and fault tolerance, providing a structured framework for adaptable quantum communication and computation protocols.

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Nagata-type automorphisms over a principal ideal domain

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Joint work with: M'hammed El Kahoui, Najoua Essamaoui

Abstract

Given a ring R and $n \geq 1$, an R -automorphism of the polynomial ring $R^{[n]}$ is said to be *tame*, in some fixed coordinate system x , if it is a composition of *affine* and *elementary* R -automorphisms. When $n = 2$ and R is a field the famous Jung-van der Kulk Theorem [1,2] asserts that all automorphisms of $A = R^{[2]}$ are tame. Such a result no longer holds if R is not a field, and Nagata [3] was the first to give an example of non-tame automorphism of $R[x_1, x_2]$, where $R = K[x_3] = K^{[1]}$ and K is a field of characteristic zero. He also conjectured that his example is even non-tame as an automorphism of $K[x_1, x_2, x_3] = K^{[3]}$. Shestakov and Umirbaev [4,5] developed a remarkable theory which allows to algorithmically recognize tame automorphisms of $K[x_1, x_2, x_3] = K^{[3]}$, where K is a field of characteristic zero. As a consequence, they positively settled the Nagata Conjecture.

An R -automorphism σ of $A = R^{[n]}$ is said to be *m-stably tame* for some $m \geq 1$, in the coordinate system x , if after the addition of m variables, say $y = y_1, \dots, y_m$, the *trivial* extension of σ to $R[x, y] = R^{[n+m]}$ defined by $\sigma(y_i) = y_i$ is a tame automorphism. In this talk, we present our main result in [6], which shows that the well-known result due to Smith [7], asserting that the Nagata automorphism is 1-stably tame, actually holds in full generality. More precisely, we show that every exponential automorphisms in two variables over a principal ideal domain is 1-stably tame in an appropriate coordinate system.

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Some Combinatorial Identities for Higher-Order Mersenne Numbers

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Abstract

Mersenne numbers, defined as $M_n = 2^n - 1$, form a fascinating integer sequence due to their unique structure. One notable generalization is the *higher-order Mersenne numbers* [3], which are defined as $M_n^{(s)} = \frac{M_{ns}}{M_s}$, $n \geq 0$, $s \geq 1$. Since M_{ns} is divisible by M_s , the ratio $\frac{M_{ns}}{M_s}$ is always an integer. In the special case when $s = 1$, we recover the Mersenne numbers: $M_n^{(1)} = M_n$.

A Toeplitz–Hessenberg matrix is an $n \times n$ matrix of the form

$$M_n(a_0; a_1, \dots, a_n) = \begin{pmatrix} a_1 & a_0 & 0 & \cdots & 0 & 0 \\ a_2 & a_1 & a_0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots & \cdots \\ a_{n-1} & a_{n-2} & a_{n-3} & \cdots & a_1 & a_0 \\ a_n & a_{n-1} & a_{n-2} & \cdots & a_2 & a_1 \end{pmatrix},$$

where $a_0 \neq 0$ and $a_k \neq 0$ for at least one $k > 0$. We investigate certain families of Toeplitz–Hessenberg determinants with $a_0 = \pm 1$ the entries of which are higher-order Mersenne numbers. For brevity, we will write $D_{\pm}(a_1, \dots, a_n)$ instead of $\det(M_n(\pm 1; a_1, \dots, a_n))$. Using the Trudi formula (see [1, 2] for more details), these determinant formulas can be rewritten as identities involving sums of products of Mersenne numbers and multinomial coefficients.

The following theorems provide the value of $D_{\pm 1}(a_1, \dots, a_n)$ for certain entries: $M_n^{(2)} = \frac{1}{3}M_{2n}$ and $M_n^{(3)} = \frac{1}{7}M_{3n}$.

Let $\{F_k\}_{k \geq 0}$ denote the Fibonacci sequence, defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$, $n \geq 2$, with initial values $F_0 = 0$ and $F_1 = 1$.

Theorem 1. Let $n \geq 1$, except where noted otherwise. Then

$$\begin{aligned} D_+(M_0^{(2)}, M_1^{(2)}, \dots, M_{n-1}^{(2)}) &= (-1)^{n-1} \sum_{k=1}^{n-1} F_{2k}, \\ D_+(M_1^{(2)}, M_2^{(2)}, \dots, M_n^{(2)}) &= n(-2)^{n-1}, \\ D_-(M_1^{(2)}, M_2^{(2)}, \dots, M_n^{(2)}) &= 2^{n-1} F_{2n}, \\ D_+(M_2^{(2)}, M_3^{(2)}, \dots, M_{n+1}^{(2)}) &= 0, \quad n \geq 3, \\ D_+(M_2^{(2)}, M_4^{(2)}, \dots, M_{2n}^{(2)}) &= 5(-4)^{n-1} F_{2n}. \end{aligned}$$

Theorem 2. Let $n \geq 1$, except where noted otherwise. Then

$$\begin{aligned} D_+(M_0^{(3)}, M_1^{(3)}, \dots, M_{n-1}^{(3)}) &= \frac{(-1)^{n-1}}{3} \sum_{k=0}^{n-1} \binom{n-1}{k} F_{4k}, \\ D_-(M_0^{(3)}, M_1^{(3)}, \dots, M_{n-1}^{(3)}) &= \frac{1}{\sqrt{53}} \left(\left(\frac{9 + \sqrt{53}}{2} \right)^{n-1} - \left(\frac{9 - \sqrt{53}}{2} \right)^{n-1} \right), \\ D_+(M_1^{(3)}, M_2^{(3)}, \dots, M_n^{(3)}) &= \frac{\sqrt{2}}{8} \left((-4 + 2\sqrt{2})^n - (-4 - 2\sqrt{2})^n \right), \\ D_-(M_1^{(3)}, M_2^{(3)}, \dots, M_n^{(3)}) &= \frac{\sqrt{17}}{34} \left((5 + \sqrt{17})^n - (5 - \sqrt{17})^n \right), \\ D_+(M_2^{(3)}, M_3^{(3)}, \dots, M_{n+1}^{(3)}) &= 0, \quad n \geq 3. \end{aligned}$$

Note that for Mersenne numbers $M_n^{(1)} = M_n$, we derived similar determinant formulas in [1].

From Theorems 1 and 2, using Trudi's formula, we obtain identities involving Mersenne numbers and multinomial coefficients $p_n(s) = \frac{(s_1 + \dots + s_n)!}{s_1! \dots s_n!}$. We will provide only the formulas that relate Mersenne and Fibonacci numbers.

Corollary 1. The following identities hold:

$$\begin{aligned} \sum_{\sigma_n=n} \left(-\frac{1}{3} \right)^{|s|} p_n(s) M_0^{s_1} M_1^{s_2} \dots M_{n-1}^{s_n} &= - \sum_{k=1}^{n-1} F_{2k}, \\ \sum_{\sigma_n=n} \frac{p_n(s)}{3^{|s|}} M_1^{s_1} M_2^{s_2} \dots M_n^{s_n} &= 2^{n-1} F_{2n}, \\ \sum_{\sigma_n=n} \left(-\frac{1}{3} \right)^{|s|} p_n(s) M_2^{s_1} M_4^{s_2} \dots M_{2n}^{s_n} &= -5 \cdot 4^{n-1} F_{2n}, \\ \sum_{\sigma_n=n} \left(-\frac{1}{7} \right)^{|s|} p_n(s) M_0^{s_1} M_1^{s_2} \dots M_{n-1}^{s_n} &= -\frac{1}{3} \sum_{k=1}^{n-1} \binom{n-1}{k} F_{4k}, \end{aligned}$$

where $|s| = s_1 + \dots + s_n$ and $\sigma_n = s_1 + 2s_2 + \dots + ns_n$ with $s_i \geq 0$.

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Groupes de classes comme modules galoisiens pour quelques extensions non abéliennes

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Joint work with: Takenori Kataoka

Résumé

La conférence s'appuie sur des travaux en commun avec Takenori Kataoka. On considère une extension G -galoisienne de corps de nombres, et son groupe Cl_K de classes comme module sur $\mathbb{Z}[G]$. Beaucoup d'information sur ce module est cachée dans les valeurs spéciales de certaines fonctions zêta et L . En particulier on peut en extraire l'ordre de la "partie moins" $Cl_K 1^-$ du groupe des classes. Mais ce n'est pas tout. On ne peut s'y attendre que la classe d'isomorphisme de ce module puisse être déduite de ces valeurs spéciales, mais comme but plus abordable beaucoup de travaux ont ciblé les idéaux de Fitting. Notre approche sera un peu différente. Nous introduisons une certaine relation d'équivalence sur l'ensemble de tous les modules finis sur $\mathbb{Z}[G]^-$, et nous présenterons une méthode pour déterminer la classe du module $Cl_K 1^-$. On trouve que dans un sens, parmi la totalité des modules fines, seulement très peu de modules peuvent se réaliser dans cette façon. Kataoka et moi avons fait cette étude pour G abélien, et maintenant nous sommes en mesure de l'étendre à quelques groupes metacycliques. Chemin faisant, il nous faut utiliser un résultat assez vieux, qui donne la classification de certains réseaux sur $\mathbb{Z}_p[G]$ pour quelques groupes G finis metacycliques; une telle classification est hors de portée pour un groupe G fini arbitraire.

On Theta Invariants, Zeta Functions and the Volume Function on Arithmetic Varieties

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Abstract

In this talk, I will introduce new invariants for Hermitian line bundles on arithmetic varieties. These invariants are used to study the arithmetic volume function.

Characterization of Extreme Cases of the Chebyshev's Bias in Number Fields

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Chebyshev's bias describes the tendency of prime numbers to favor certain residue classes over others. First observed by Chebyshev in the case of primes modulo 4, this phenomenon was later studied rigorously by Rubinstein and Sarnak [1], who, under some hypotheses on the zeros of Dirichlet L -functions, developed a framework to quantify it modulo any integer q .

In this talk, we will present recent developments on Chebyshev's bias in number fields, starting from the unconditional results of Fiorilli and Jouve [1], who established that, unlike in $\mathbb{Q}$, extreme biases can occur unconditionally in certain number fields. We will then discuss our work, in which we extend their unconditional results and construct new instances of explicit Galois extensions with extreme Chebyshev's bias.

Building on this, we will present our latest results, where under the assumptions of the Generalized Riemann Hypothesis (GRH), the Artin Conjecture, and a Linear Independence hypothesis on Artin L -function zeros, we establish a full characterization of when such extreme biases appear. This characterization leads to a natural **trichotomy**: either prime biases disappear, remain in the classical range, or become completely one-sided.

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Structured Distributions of Class Groups of some Real Pure Quartic Fields

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Joint work with: Mohammed TAOUS

Abstract

Let $K = \mathbb{Q}(\sqrt[4]{pd^2})$ be a real pure quartic number field and $k = \mathbb{Q}(\sqrt{p})$ its unique real quadratic subfield, where $p \equiv 1 \pmod{8}$ is an odd prime number and d is a square-free positive integer such that $(p, d) = 1$. In this talk, we explore the distributions of certain 2-class groups associated with K , demonstrating that their distribution exhibits an orderly and deterministic pattern. Our results reveal a structured and deterministic behavior in their statistical distribution. Using a combination of analytic methods and computational techniques, we provide both theoretical insights and numerical evidence. The computations, carried out in the Pari/GP algebra system, serve to compare the observed distributions with theoretical predictions, highlighting key patterns and confirming conjectural expectations.

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On the coclass and coclass Graphs

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Joint work with: M. Sahmoudi and M. Taous

Abstract

This communication explores the application of coclass graphs in the classification of finite p -groups, a fundamental topic in algebraic number theory. The concept of coclass, introduced by Leedham-Green and Newman, organizes p -groups into structured families, offering a systematic approach to their enumeration. We present key properties of coclass graphs, including their self-similar structure and finite family distribution, and highlight their role in studying some results in some number theory problems. Computational methods, based on algorithms developed by Eick, are also discussed, demonstrating how these tools facilitate the exploration of complex p -group structures.

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On the transcendence of some continued fractions

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Joint work with: Sarra Ahallal

Abstract

The theory of transcendental numbers has a long history. We know since J. Liouville in 1844 that the very rapidly converging sequences of rational numbers provide examples of transcendental numbers. He precisely showed that a real number admitting very good rational approximations can not be algebraic. In the present In the present paper, we give sufficient conditions on the elements of continued fractions A and B which will assure us that the continued fraction A^B is a transcendental number. With the same condition, we establish a transcendental measure of A^B .

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Constructing infinite families of number fields with given indices from quintinomials

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Joint work with: Lhoussain El Fadil

Abstract

In this presentation, for any rational prime p and for a fixed positive integer ν_p , we provide infinite families of number fields defined by quintinomials satisfying $\nu_p(i(K)) = \nu_p$.

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Attaques sur le problème du logarithme discret (DLP et ECDLP)

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Joint work with: Khadija BOUZKOURA

Résumé

Le problème du logarithme discret sur les courbes elliptiques (ECDLP) est essentiel en cryptographie, mais plusieurs attaques menacent sa sécurité. Cette communication explore les principales méthodes d'attaque, notamment rho de Pollard, Pohlig-Hellman, MOV et la descente elliptique, ainsi que l'impact de l'algorithme de Shor en cryptographie quantique. Enfin, nous présenterons des stratégies pour renforcer la sécurité des systèmes ECC face à ces menaces.

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Asymptotic Evaluation of Some Arithmetic Functions

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Joint work with: Professor Abdelaziz Raouj

Abstract. Generally, the Asymptotic evaluation of arithmetic functions is a classical topic in number theory, where we study the behavior of the sum $\sum_{n \leq x} f(n)$, where f is an arithmetic function and $x > 0$. The purpose in the required sum is to give a good error term. The techniques used in a such problem are depend one the nature of $f(n)$. When dealing on the short sums of arithmetic functions, we are typically interested in the sums of the form :

$$\sum_{x-y < n \leq x} f(n), \quad \text{where } 0 < y \leq x$$

Researchers are focused on the interval $]x - y, x]$ in order to minimize it and in same time to give a good error term.

In this topic, we will present our recent results in this area. The tools used are analytics and combinatorial methods.

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The Freeness Property for Locally Nilpotent Derivations on Danielewski Varieties

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Joint work with: M'hammed EL Kahoui, Mustapha Ouali

Abstract. Locally nilpotent derivations play a fundamental role in understanding automorphisms of polynomial algebras. While Rentschler [7] completely classified locally nilpotent derivations of the polynomial ring $K^{[2]}$, where K is a field of characteristic zero, the structure of locally nilpotent derivations for $K^{[3]}$ remains far less understood despite significant contributions from Miyanishi [6], Bonnet [2] and Kaliman [5].

As an attempt to explore this gap, Freudenburg [4] introduced the *freeness property* for a nonzero locally nilpotent derivation ζ of $A = K^{[3]}$, which asserts that A is free as a module over the ring of constants of ζ , with basis that satisfies a certain graded structure. He also conjectured that nonzero locally nilpotent K -derivations of $K^{[3]}$ have the freeness property. Recently, Ben Khaddah, El Kahoui and Ouali [1] proved that the freeness property holds for nonzero locally nilpotent derivations of $R^{[2]}$, where R is a principal ideal domain containing $\mathbb{Q}$. This settles Freudenburg freeness Conjecture for locally nilpotent K -derivations of rank at most two. The proof of this result revealed a close relationship between the freeness property and the Danielewski algebras and varieties as well as their generalizations.

In this talk, we explore the freeness property within the context of Danielewski varieties and present constructions and characterizations that offer insights into the nature and the implication of this property for locally nilpotent derivations. Some of the results we present can be found in [3].

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The finitude of tamely ramified pro- p -extensions of number fields with cyclic p -class groups

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Abstract

Let p be an odd prime and F be a number field whose p -class group is cyclic. Let $F_{\{q\}}$ be the maximal pro- p extension of F which is unramified outside a single non- p -adic prime ideal q of F . In this work, we study the finitude of the Galois group $G_{\{q\}}(F)$ of $F_{\{q\}}$ over F . We prove that $G_{\{q\}}(F)$ is finite for the majority of q 's such that the generator rank of $G_{\{q\}}(F)$ is two, provided that for $p = 3$, F is not a complex quartic field containing the primitive third roots of unity.

On the variant $n! = P(x)$ of the Brocard-Ramanujan Diophantine equation with k -free integers

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Abstract

Let $P \in \mathbb{Z}[X]$ be a polynomial with $P(0) = 0$. We investigate the Diophantine equation $n! = P(x)$, a variant of the Brocard-Ramanujan equation. Our main result establishes that for any pair (k, l) of positive integers, this equation has only finitely many integer solutions (n, x) under the assumption that x is either k -free or has fewer than l prime divisors. Moreover, we provide a complete characterization of integer solutions when x is square-free or a prime power.

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Counting Points on an Elliptic Curve over Finite Fields

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Joint work with: Pr: M. SAHMOUDI et Pr: A. CHILLALI

Abstract

In this presentation, we introduce Hasse's theorem as well as Schoof's algorithm. This algorithm allows us to compute the number of points on an elliptic curve E over a finite field $\mathbb{F}_q$, where q is a power of prime number. The complexity of this algorithm is $O(\log^8 q)$.

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On the structure of the unramified abelian Iwasawa module of some cyclic number fields

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Abstract

For a prime number p , we consider the maximal unramified pro- p -extension over the cyclotomic $\mathbb{Z}_p$ -extension of a cyclic number field of degree p , which is the union of p -class field towers along the $\mathbb{Z}_p$ -extension. We give a general formula on the generator rank of the Galois group of such maximal unramified pro- p -extension. As an application, we give the structure of the unramified abelian Iwasawa module of some cyclic number fields.

Signature maps in number fields.

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Abstract

Let F be a number field which is not totally imaginary. For any real place v , let $i_v : F \rightarrow \mathbf{R}$ denote the corresponding real embedding. The natural signature maps $\text{sgn}_v : F^\bullet \rightarrow \mathbb{Z}/2$ (where $\text{sgn}_v(x) = 0$ or 1 according to whether $i_v(x) > 0$ or not) defined on nonzero elements of F give rise to the following surjective signature map:

$$\begin{aligned} F^\bullet &\longrightarrow \bigoplus_{v \text{ real}} \mathbb{Z}/2 \\ x &\longmapsto (\text{sgn}_v(x))_{v \text{ real}} \end{aligned}$$

My lecture, a joint work with Jilali Assim, concerns restrictions of the above signature map to some subgroups such as generalized Tate kernels introduced by Manfred Kolster.

Des entiers vers les polynômes

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Joint work with: Arnaud Bodin, Pierre Dèbes, Joachim Koenig.

Résumé

Dans cet exposé, je vais présenter quelques résultats connus pour les entiers (par exemple le théorème de progression arithmétique de Dirichlet) et beaucoup d'autres sous forme de questions ouvertes (par exemple l'hypothèse (H) de Schinzel-Sierpiński, le problème des nombres premiers jumeaux, le problème de Goldbach, etc.). Des analogues de ces résultats et questions pour les polynômes ont été principalement étudiés dans des papiers avec A. Bodin, P. Dèbes et J. Koenig : [BDN1], [BDN2] et [BDKN].

1. L'hypothèse (H)

Soient $P_1(x), \dots, P_s(x)$ des polynômes irréductibles de $\mathbb{Z}[x]$. Supposons qu'aucun nombre premier ne divise le produit $\prod_{i=1}^s P_i(n)$ pour tout $n \in \mathbb{Z}$. Alors il existe une infinité de $n \in \mathbb{Z}$ tels que $P_1(n), \dots, P_s(n)$ soient tous des nombres premiers.

• Conséquence 1 :

Une infinité de nombres premiers de la forme $n^2 + 1$.

• Conséquence 2 :

Une infinité de nombres premiers jumeaux (c'est-à-dire une infinité de nombres premiers p tels que $p + 2$ soit aussi premier).

• Seul cas connu (théorème de Dirichlet) :

Soit $P(x) = ax + b$. Si $\gcd(a, b) = 1$, alors il existe une infinité de $n \in \mathbb{Z}$ tels que $an + b$ soit premier.

2. Des entiers vers les polynômes

L'hypothèse (H) pour les entiers VS l'hypothèse (H) pour les polynômes :

$$\begin{aligned} \mathbb{Z} &\longrightarrow Z = \mathbb{Z}[y_1, \dots, y_m] = \mathbb{Z}[\underline{y}], \\ P(x) \in \mathbb{Z}[x] &\longrightarrow P(x, \underline{y}) \in \mathbb{Z}[x] = \mathbb{Z}[x, \underline{y}], \\ P(n) &\longrightarrow P(N(\underline{y}), \underline{y}) \in Z = \mathbb{Z}[\underline{y}], \\ P(n) \text{ premier} &\longrightarrow P(N(\underline{y}), \underline{y}) \text{ irréductible dans } \mathbb{Z}[\underline{y}], \\ \text{infinité} &\longrightarrow \text{Zariski dense.} \end{aligned}$$

Version simple.

Soit $P(x, y) \in \mathbb{Z}[x, y]$ un polynôme irréductible de degré $\deg_x P \geq 1$. Alors il existe plusieurs polynômes $N(y) \in \mathbb{Z}[y]$ tels que $P(N(y), y)$ soit irréductible dans $\mathbb{Z}[y]$.

Version plus générale.

Soit R un anneau factoriel et $Z = R[y_1, \dots, y_m] = R[\underline{y}]$ avec $m \geq 0$,

$K = \text{Frac}(R)$ ayant une formule du produit et imparfait si $\text{car}(K) = p > 0$ (c'est-à-dire $K \neq K^p$).

Théorème.

Soient $P_1, \dots, P_s \in R[x, \underline{y}]$ des polynômes irréductibles de degré $\deg_x P_i \geq 1$. Pour tout $\underline{d} = (d_1, \dots, d_n) \in (\mathbb{N}^*)^n$ tel que $d_1 + \dots + d_n \geq \max_{1 \leq i \leq s} \deg_{\underline{y}}(P_i) + 2$, l'ensemble des $N(\underline{y})$ tels que les $P_i(N(\underline{y}), \underline{y})$ soient irréductibles dans $R[\underline{y}]$ est Zariski dense dans

$$\left\{ M \in R[\underline{y}] \mid \deg_{y_i}(M) \leq d_i \text{ pour tout } i \right\}.$$

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The last decade of the RSA cryptosystem

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Abstract

NIST has recently released a publication related to the transition to Post-Quantum Cryptography which specifies that most of the public key classical cryptosystems, especially RSA, will be officially deprecated by 2030 and banned after 2035. In this talk, I will present the alternative post quantum cryptosystems, review the limits of the main cryptanalytical attacks on RSA, and present two new variants of RSA based on the cubic Pell curve.

Sur certaines extensions abéliennes de corps locaux en égale caractéristique

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Résumé

Nous considérons les points de torsion de certaines actions provenant de modules formels de Drinfeld. Nous montrons qu'ils engendrent des extensions de Lubin-Tate. Nous discuterons le lien avec l'application d'Artin.

A Connection between the Milnor K_2 Group and the Shafarevich-Tate Group

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Abstract

Let $p \equiv 1 \pmod{8}$ be a prime and E_p the elliptic curve defined by $y^2 = x^3 - p^2x$. Denote by $\mathcal{O}_F$ the ring of integers of $\mathbb{Q}(\sqrt{p})$. We establish a connection between the Milnor K_2 group of $\mathcal{O}_F$ and the Shafarevich-Tate group of E_p . This demonstrates that in the case of congruent number elliptic curves, the Birch and Swinnerton-Dyer conjecture is linked to the Birch-Tate conjecture. Our results rely on investigations into quadratic forms, L -values of elliptic curves, and the interplay between ideal class groups and Milnor K_2 groups.

Some Methods in analytic and Probabilistic Number Theory

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Résumé

Le but est de présenter quelques approches variées en théorie analytique et probabiliste des nombres. Des exemples illustrent leur efficacité et mettent aussi en évidence les limites des techniques utilisées.

Abstract

The main objective of this talk is to present some varied approaches in analytic and probabilistic number theory. Examples will illustrate their effectiveness and highlight the limitations of the techniques used.

On the Iwasawa λ -invariant of some imaginary number fields

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Abstract

Let $K = \mathbb{Q}(\sqrt{q\varepsilon\sqrt{d}}, \sqrt{-1})$ be an imaginary number field, where $d \equiv -q \equiv 5 \pmod{8}$ are primes and ε is the fundamental unit of his quadratic subfield $k = \mathbb{Q}(\sqrt{d})$. In this work, we are interested to calculate the λ -invariant of the cyclotomic $\mathbb{Z}_2$ -extension K_∞/K , furthermore, we determine the structure of the the Iwasawa module of K .

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Classes de Steinitz d'extensions galoisiennes non ramifiées (Steinitz classes of unramified Galois extensions)

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Joint work with: Ayoub Al Uartassi and Mohammed Taous

Résumé

Soient K un corps de nombres et Cl_K son groupe de classes. Soit Γ un groupe fini. On désigne par $R_{nr}(K, \Gamma)$ le sous-ensemble de Cl_K formé par les classes réalisables comme classes de Steinitz d'extensions galoisiennes de K , non ramifiées aux places finies de K et ayant un groupe de Galois isomorphe à Γ . Si Γ est abélien et le nombre de classes de K au sens restreint est premier avec l'ordre de Γ , alors $R_{nr}(K, \Gamma) = \emptyset$. Dans cet exposé, pour Γ quelconque on considère l'ensemble $R'_{nr}(K, \Gamma) := \{1\} \cup R_{nr}(K, \Gamma)$. On prouve que $R'_{nr}(K, \mathbb{Z}/2\mathbb{Z})$ est un sous-groupe de $Cl_{K,2} := \{c \in Cl_K | c^2 = 1\}$; de plus il est égal à $Cl_{K,2}$ sous une certaine hypothèse sur K . En utilisant ce résultat, on montre que $R'_{nr}(K, \Gamma)$ est un sous-groupe de $R'_{nr}(K, \mathbb{Z}/2\mathbb{Z})$ si Γ est d'ordre impair, ou bien a un 2-sous-groupe de Sylow non cyclique (par exemple Γ un 2-groupe non abélien, $\Gamma = S_n$ ou A_n , avec $n \geq 4$), ou bien a un 2-sous-groupe de Sylow cyclique et normal (par exemple Γ nilpotent d'ordre pair).

Abstract

Let K be a number field and Cl_K its class group. Let Γ be a finite group. We denote by $R_{nr}(K, \Gamma)$ the subset of Cl_K formed by the classes which are realizable as Steinitz classes of Galois extensions of K , unramified at the finite places of K and having a Galois group isomorphic to Γ . If Γ is abelian and the narrow class number of K is prime to the order of Γ , then $R_{nr}(K, \Gamma) = \emptyset$. In the present talk, for any Γ we consider the set $R'_{nr}(K, \Gamma) := \{1\} \cup R_{nr}(K, \Gamma)$. We prove that $R'_{nr}(K, \mathbb{Z}/2\mathbb{Z})$ is a subgroup of $Cl_{K,2} := \{c \in Cl_K | c^2 = 1\}$; furthermore, it is equal to $Cl_{K,2}$ under a certain assumption on K . Using this result we show that $R'_{nr}(K, \Gamma)$ is a subgroup of $R'_{nr}(K, \mathbb{Z}/2\mathbb{Z})$ if Γ either has odd order, or has a noncyclic 2-Sylow subgroup (for instance Γ a nonabelian 2-group, $\Gamma = S_n$ or A_n , with $n \geq 4$), or has a normal cyclic 2-Sylow subgroup (for instance Γ nilpotent having even order).

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$\mathbb{Z}$ -bases and $\mathbb{Z}[1/2]$ -bases for Washington's cyclotomic units of real cyclotomic fields and totally deployed fields

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Abstract

Given an abelian number field $\mathbb{K}$, one may construct the cyclotomic units of $\mathbb{K}$. Those units are intertwined with arithmetical objects such as the p -adic Zeta function - through Iwasawa's theory - and the ideal class group.

Let $\mathbf{Was}(\mathbb{K})$ refers to the group of Washington's cyclotomic units of $\mathbb{K}$ and let $\mathbf{Z}(\mathbb{K})$ be the group of roots of unity lying in $\mathbb{K}$. We will deal with families of generators with minimal cardinality - we call such families bases - of the free abelian group $\mathbf{Was}(\mathbb{K})/\mathbf{Z}(\mathbb{K})$. If $\mathbb{K}$ is a totally deployed abelian number field, that is $\mathbb{K} = \mathbb{K}_1 \dots \mathbb{K}_r$ with $\mathbb{K}_i \subset \mathbb{Q}(\zeta_{p_i^{e_i}})$, we provide $\mathbb{Z}[1/2]$ -bases of $\mathbf{Was}(\mathbb{K})/\mathbf{Z}(\mathbb{K}) \otimes \mathbb{Z}[1/2]$.

To this aim, we consider a family of elements of $\mathbb{K}$ that has $\text{rg}_{\mathbb{Z}}(\mathbf{Was}(\mathbb{K})) = r_1 + r_2 - 1$ elements (with usual notation) and that generates a direct factor of $\mathbf{Was}(\mathbb{Q}(\zeta_n))/\mathbf{Z}(\mathbb{Q}(\zeta_n)) \otimes \mathbb{Z}[1/2]$. It is not hard to see that this property makes this family generate $\mathbf{Was}(\mathbb{K})/\mathbf{Z}(\mathbb{K}) \otimes \mathbb{Z}[1/2]$ so that this family is a basis. More precisely, we construct a basis of $\mathbf{Was}(\mathbb{K})/\mathbf{Z}(\mathbb{K}) \otimes \mathbb{Z}[1/2]$ that can be completed with Kucera's basis to form a basis of $\mathbf{Was}(\mathbb{Q}(\zeta_n))/\mathbf{Z}(\mathbb{Q}(\zeta_n)) \otimes \mathbb{Z}[1/2]$. This idea has already been used in [4] and [1].

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An infinite family of trace-free monogenic trinomials

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Joint work with: Daniel C. Mayer.

Abstract

For an infinite family of monogenic trinomials $P(X) = X^3 \pm 3rbX - b \in \mathbb{Z}[X]$, arithmetical invariants of the cubic number field $L = \mathbb{Q}(\theta)$, generated by a zero θ of $P(X)$, and of its Galois closure $N = L(\sqrt{d_L})$ are determined. The conductor f of the cyclic cubic relative extension N/K , where $K = \mathbb{Q}(\sqrt{d_L})$ denotes the unique quadratic subfield of N , is proved to be of the form $3^e b$ with $e \in \{1, 2\}$, which admits statements concerning primitive ambiguous principal ideals, lattice minima, and independent units in L . The number m of non-isomorphic cubic fields $L_1, \dots, L_m$ sharing a common discriminant $d_{L_i} = d_L$ with L is determined.

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Capitulation in the unramified extensions of the field $\mathbb{K} = \mathbb{Q}(\sqrt{2p\varepsilon_0\sqrt{\ell}})$ where $\ell \equiv 1 \pmod{8}$ and $p \equiv 1 \pmod{4}$

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Joint work with: A. Zekhnini and A. AZIZI.

Abstract

Given a real cyclic quartic number field $\mathbb{K} = \mathbb{Q}(\sqrt{2p\varepsilon_0\sqrt{\ell}})$ where p and ℓ are two positive prime integers satisfying $\ell \equiv 1 \pmod{8}$ and $p \equiv 1 \pmod{4}$. Let ε_0 be the fundamental unit of the unique quadratic subfield $k = \mathbb{Q}(\sqrt{\ell})$ of $\mathbb{K}$. Our purpose is to study the capitulation of ideal classes of class-group isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ of $\mathbb{K}$ in the quadratic subfields of the first Hilbert class field of $\mathbb{K}$.

Keywords. Real cyclic quartic number field, 2-class group, unramified extensions, capitulation.

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Représentation des entiers par des familles de formes binaires

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Résumé

L'estimation asymptotique du nombre d'entiers qui sont sommes de deux carrés fait intervenir la constante de Landau–Ramanujan. Ce résultat a été étendu par Paul Bernays en 1912 aux formes binaires quadratiques de discriminant non carré à la place de $X^2 + Y^2$. De nombreux travaux ont été consacrés à l'étude asymptotique du nombre d'entiers représentés par une forme binaire de degré au moins 3, jusque quand Cam Stewart et Stanley Yao Xiao ont complètement résolu la question dans un article publié en 2019.

Dans cet exposé nous nous intéressons à des familles de formes binaires. Le premier exemple que nous avons étudié dans un article avec Claude Levesque et Etienne Fouvry est celui de la famille des formes cyclotomiques. Ensuite, dans une série d'articles avec Étienne Fouvry, nous avons considéré des familles plus générales de formes binaires. Nos résultats récents reposent sur des minoration de formes linéaires de logarithmes et sur l'étude des homographies entre deux formes binaires.

On the Hilbert 2-class field of some real quadratic number fields

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Joint work with: B. AABOUN and A. BEN AMAR,

Abstract

For some real quadratic numbers fields $\mathbf{k}$, we aim, in this note, to give necessary and sufficient criteria for the 2-class group of $\mathbf{k}_2^{(1)}$, the first Hilbert 2-class field of $\mathbf{k}$, to be cyclic.

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